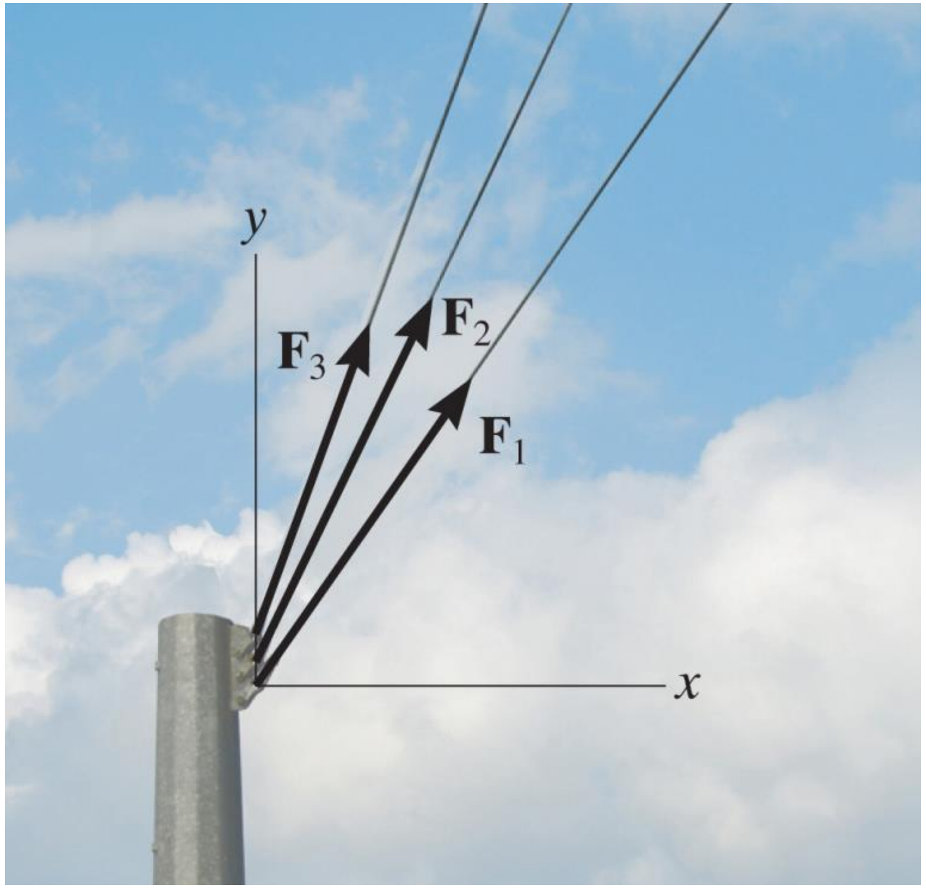
ENED1120: Foundations of Engineering Design Thinking II

Mechanics I: Vectors, Forces, and Moments

Outline

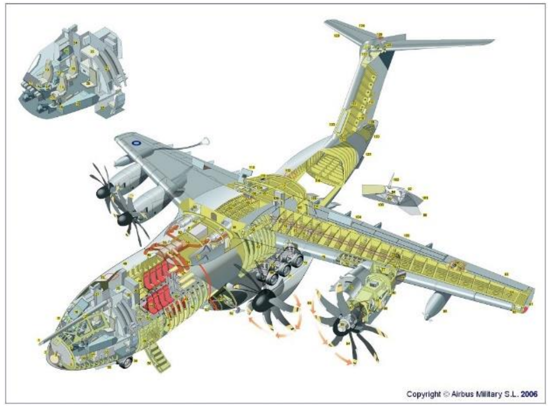
- Announcements
- Mechanics 1
  - Mechanics Overview
  - Forces and Vectors
  - Free Body Diagram
  - Systems of Forces
  - Example

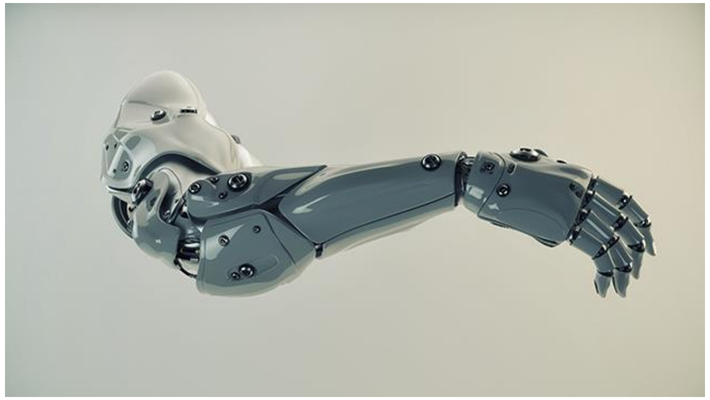
Mechanics I:
Vectors, Forces, and Moments



Why Study Mechanics?

- Mechanics is the study of how a system behaves when acted on by forces





Machines are a Central Theme

- Machines include everything from mechanical pencils to automobiles to the human body.
- Machines are designed to fulfill some function (design).
- Machines must be constructed (manufacturing).
- Machines have to be made from something (materials).



More About Machines –(topic areas found here)

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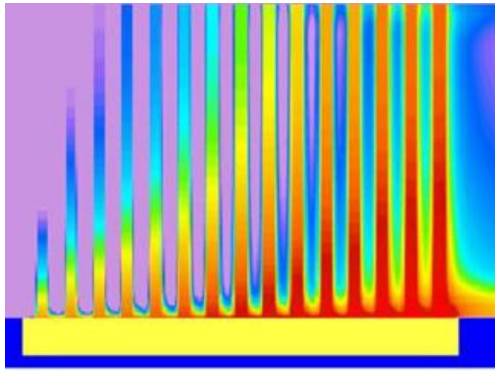
- A machine's motion must be understood (dynamics) and controlled (controls).
- Machines may be surrounded by air or water (fluids).
- Machines require power (thermal sciences).
- Machines create heat (heat transfer).



Mechanics

- The study of the mechanics of rigid bodies is divided into two branches:

Statics: system is not accelerating

Dynamics: system is accelerating

Newton’s Laws of Motion

- The foundation of classical mechanics is based on Newton’s Laws of Motion:
  1. An object at rest will stay at rest or an object in motion will stay in motion **unless acted on by a force**
  2. The **sum of the forces** acting on an object is equal to the mass of the object multiplied by its **acceleration** → $\sum F = m_{object} * a_{object}$
  3. When one object exerts a force on a second object, that second object exerts a force **equal in magnitude and opposite in direction** on the first object

Representing Forces → Vectors

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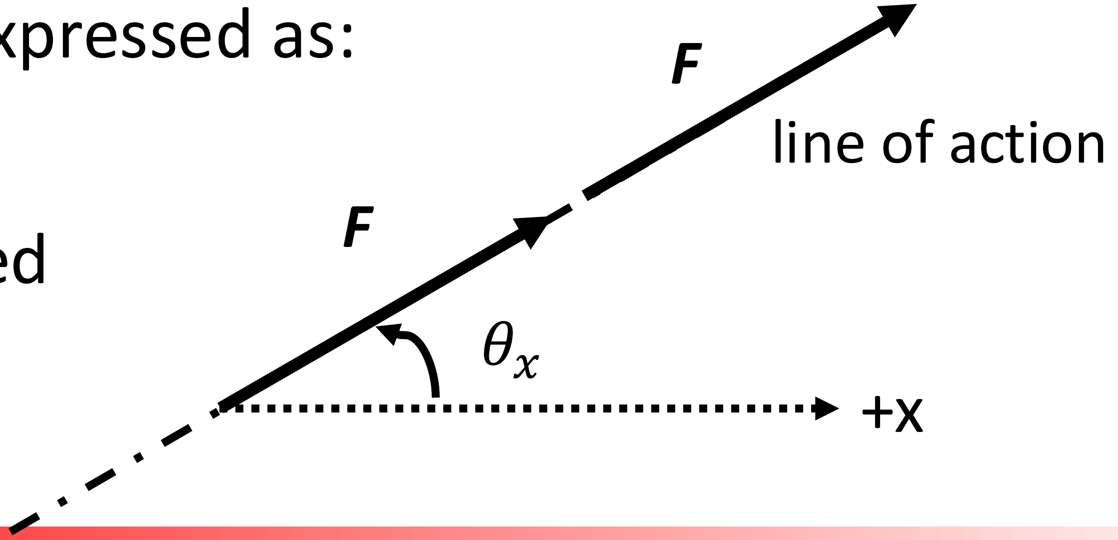
- We model forces using ***vectors***
- Vectors are mathematical quantities with two aspects:
  - **Magnitude** – the size of the vector (Newtons (N) or Pound Force (lb))
  - **Direction** – which way the vector is pointing (Degrees (°) or Radians (rad) with respect to the positive x-axis)
- These two elements are typically expressed as:

$\vec{\boldsymbol{F}} = F\measuredangle\theta_x \qquad \vec{\boldsymbol{F}} = F\measuredangle\theta_y$

- A vector has a **line of action** (defined by the direction) along which the vector can be moved



Free Body Diagrams

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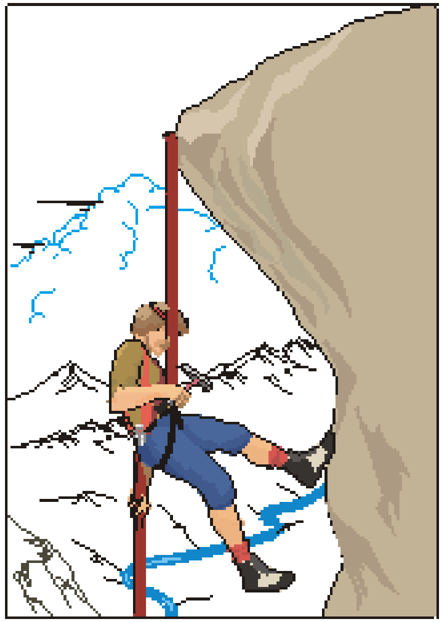
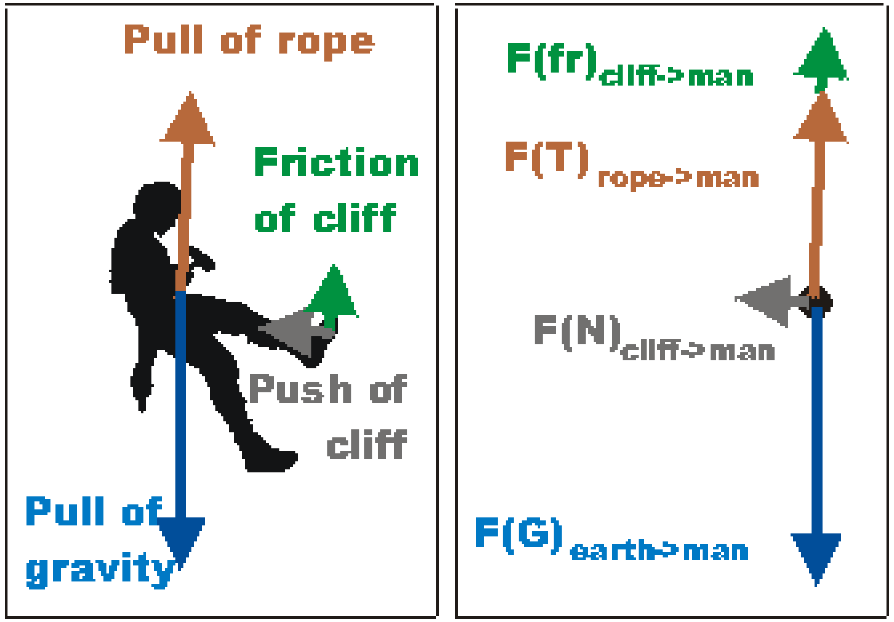
- Free-body diagrams are diagrams that show the relative **magnitude** and **direction** of all forces acting upon an object in a given situation.



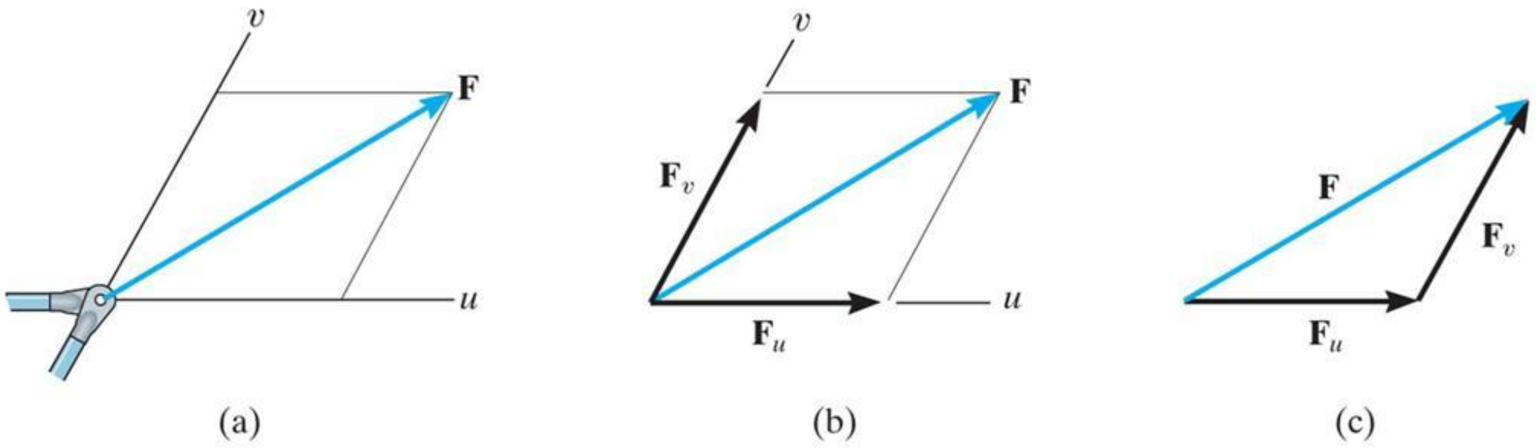
Problem



Free Body Diagram of Mass

RESOLUTION OF A VECTOR

"Resolution" of a vector is breaking up a vector into components.



It is kind of like using the parallelogram law in reverse.

Representing Forces

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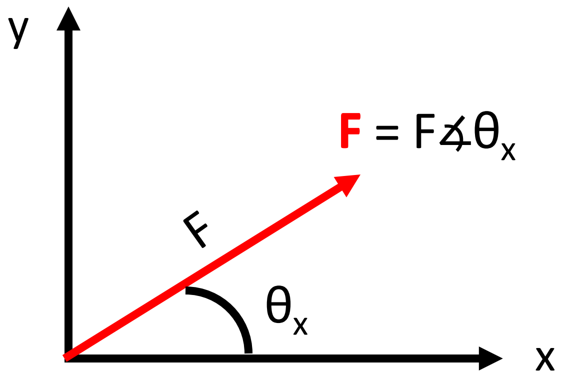
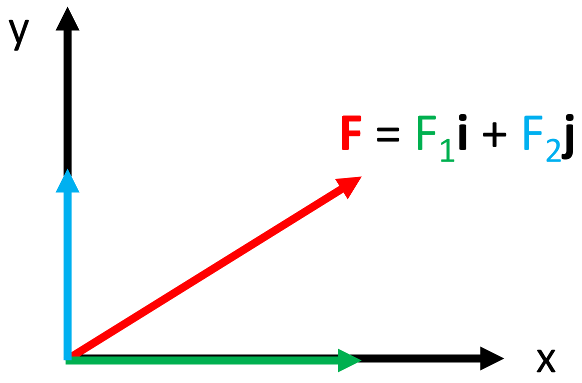
- Two ways to represent a vector:



F is the magnitude of the force

θx is the angle of the force with respect to the positive x-axis



i is the unit vector along the x-axis
j is the unit vector along the y-axis

F_1 and F_2 are the x and y components of the force vector **F**, respectively

Polar Form

$$F = \sqrt{F_1^2 + F_2^2}$$

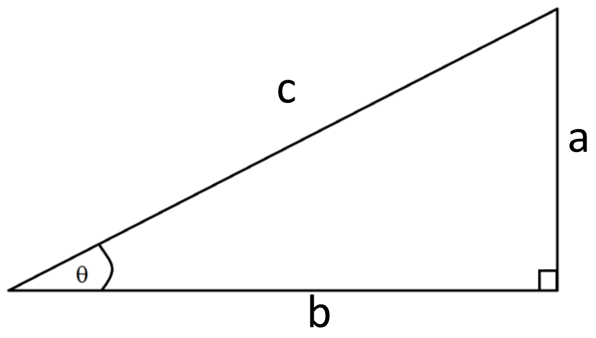
$$\theta_x = \tan^{-1}\left(\frac{F_2}{F_1}\right)$$

Cartesian (Rectangular) Form

$$F_1 = F^*\cos(\theta_x)$$

$$F_2 = F^*\sin(\theta_x)$$

Review: Triangle Trigonometry



Write equations for

Sin $\theta =$

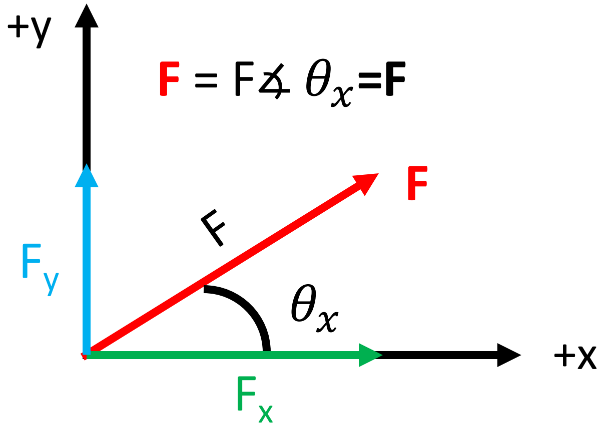
Cos $\theta =$

a =

b =

Representing Forces

- Determining the force quadrant and corresponding θ_x

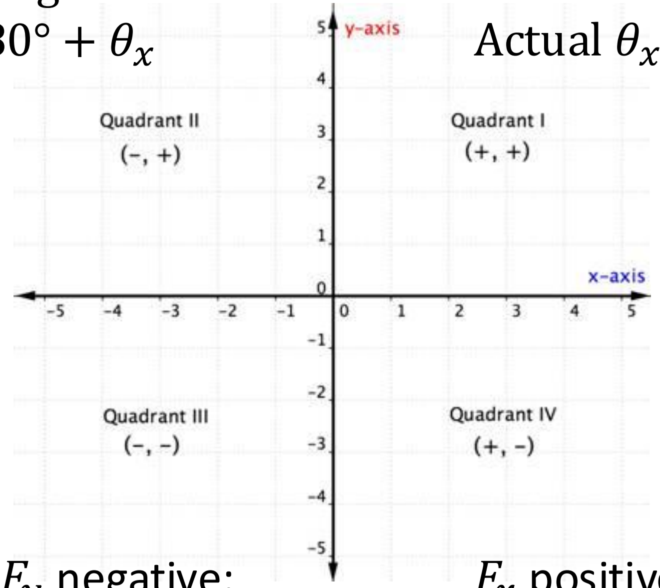


$$\theta_x = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

F_x negative and F_y positive:
Calculator: θ_x negative
Actual $\theta_x = 180° + \theta_x$

F_x positive and F_y positive:
Calculator: θ_x positive
Actual $\theta_x = \theta_x$

If you're over here, add 180º to your angle!



F_x negative and F_y negative:
Calculator: θ_x positive
Actual $\theta_x = 180° + \theta_x$

F_x positive and F_y negative:
Calculator: θ_x negative
Actual $\theta_x = \theta_x$

Forces Acting on a System

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- When considering a system experiencing forces, there are three possible configurations:
  1. **Colinear forces:** forces acting along the same line of action

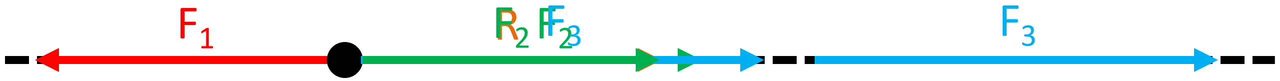
When forces are colinear, the resulting force is simply the sum of the individual forces

$$R = -F_1 + F_2 + F_3$$

Forces Acting on a System

- When considering a system experiencing forces, there are three possible configurations:
  1. **Colinear forces:** forces acting along the same line of action

     Forces can be translated along their line of action, which means we can simplify this system to show the forces all acting at the same point:



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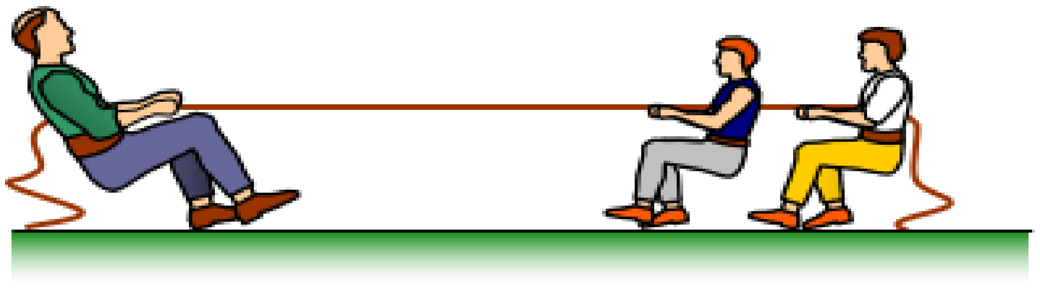

Figure 4

Forces Acting on a System

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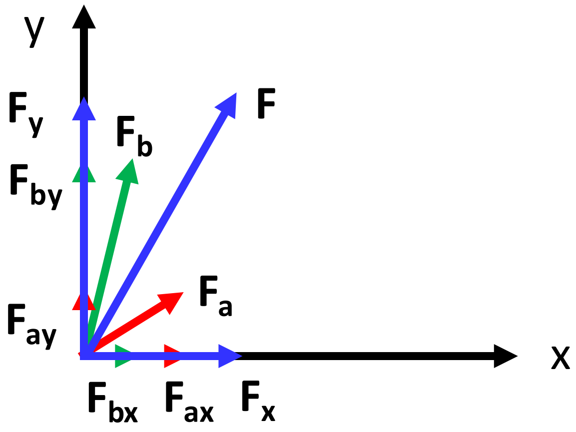
- When considering a system experiencing forces, there are three possible configurations:
  2. **Concurrent forces:** forces whose lines of action (i.e. direction) pass through the same point

     When forces are concurrent, resolve the forces into orthogonal components (i.e. x and y) and solve the resulting colinear problems



Resolve X:

$$F_{ax} = F_a \cos(\theta_a)$$
$$F_{bx} = F_b \cos(\theta_b)$$

$$F_x = F_{ax} + F_{bx}$$

Resolve Y:

$$F_{ay} = F_a \sin(\theta_a)$$
$$F_{by} = F_b \sin(\theta_b)$$

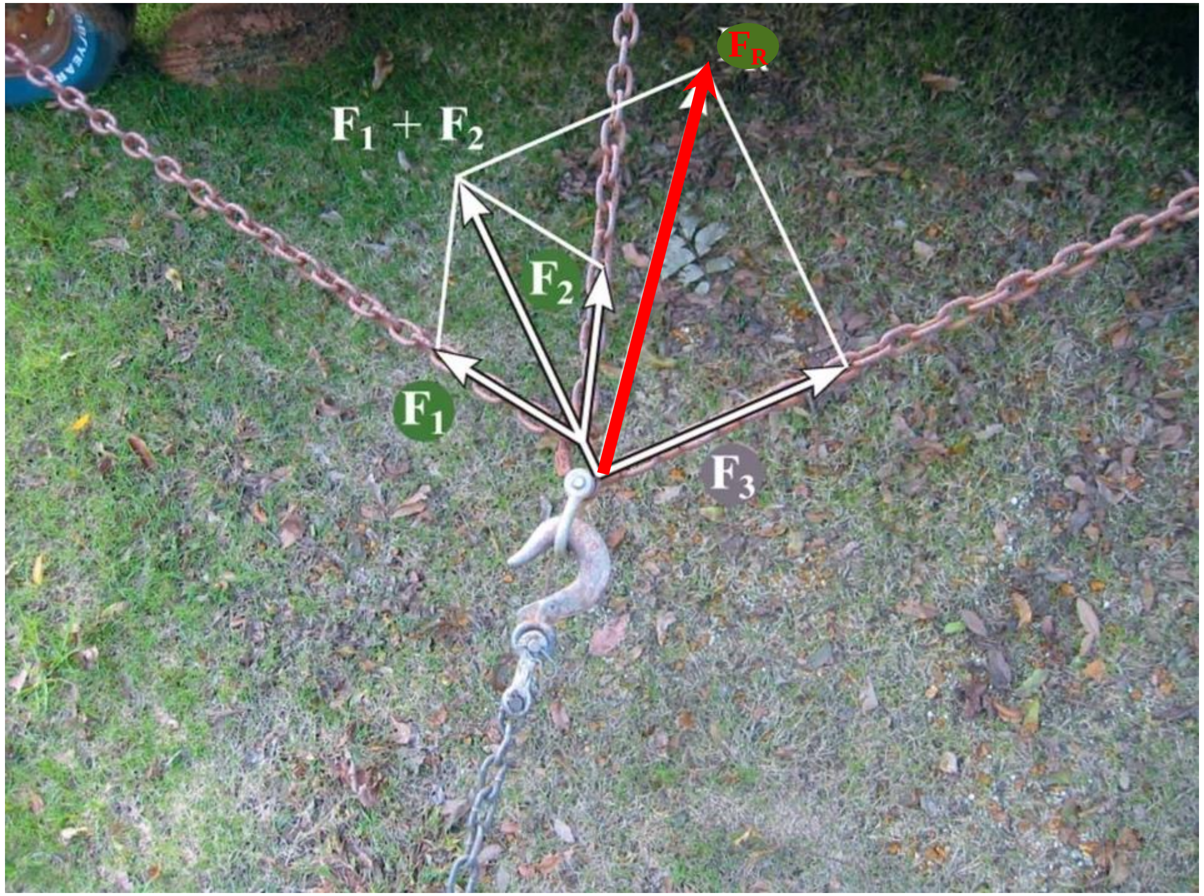
$$F_y = F_{ay} + F_{by}$$

Resultant Force:

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Concurrent Forces Example



There are three concurrent forces acting on the hook due to the chains.

We need to decide if the hook will fail (bend or break).

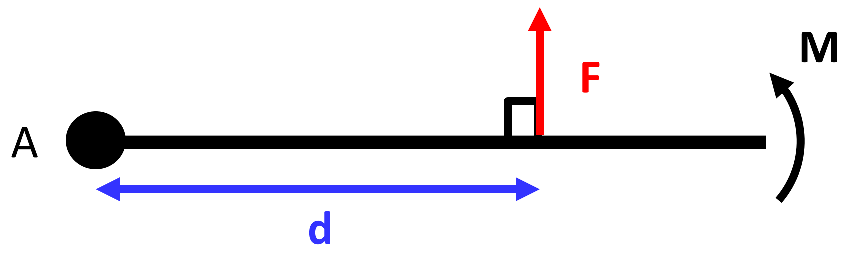
To do this, we need to know the resultant or total force acting on the hook as a result of the three chains.

Forces Acting on a System

- When considering a system experiencing forces, there are three possible configurations:

3. **Coplanar forces:** forces whose lines of action (i.e. direction) all lie in the same plane

   When forces are coplanar, attempt to shift them along their line of action to a location where all forces are acting on a line (moment arm), and compute the moments (rotational force) of the forces



$$\mathbf{M} = \pm(F * d)$$

M = + → counterclockwise rotation
M = - → clockwise rotation

Forces Acting on a System

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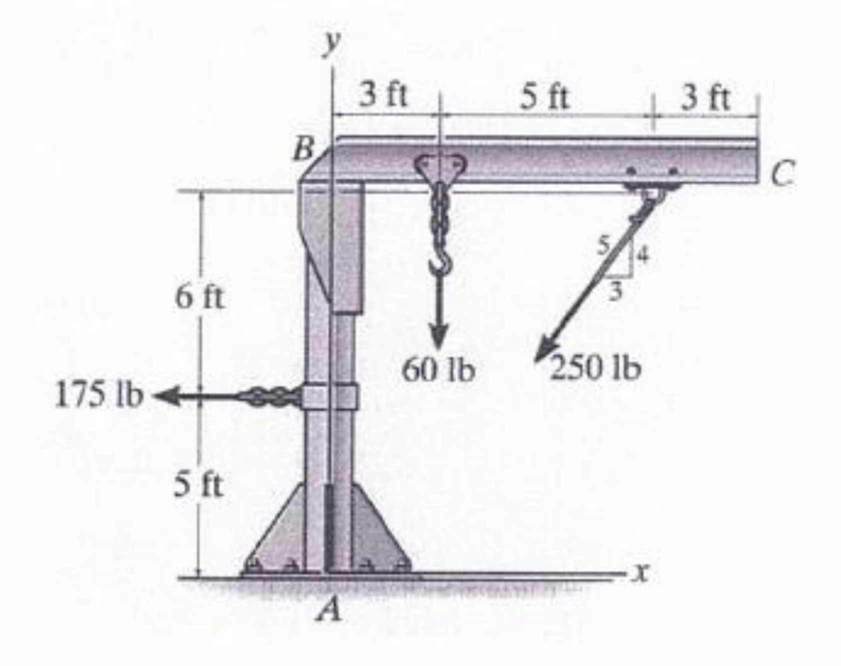
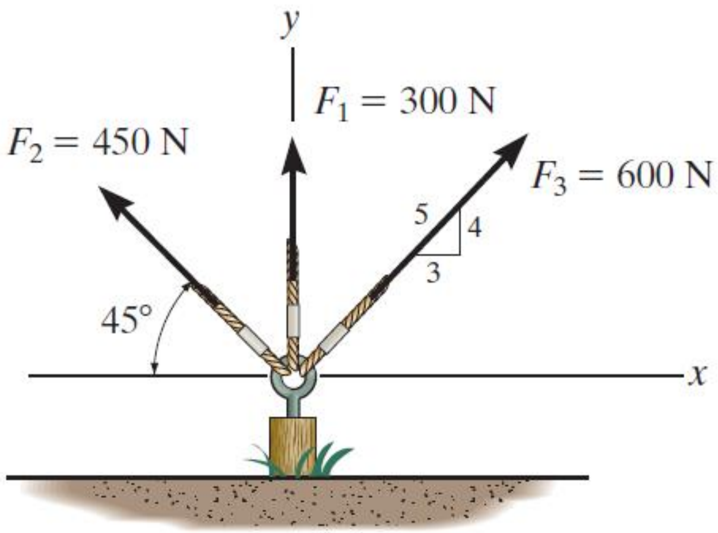


Problem Presentation Method - Mechanics

Section	Content
Problem Statement	Restate the problem in your own words, being sure to clearly identify what information was given (e.g., forces, angles, distances), making sure to include units.
Diagram	Create a diagram showing the system, the forces acting on the system, and the location of the forces.
Theory	$F_x = F\cos(\theta) \quad F_y = F\sin(\theta) \quad F = \sqrt{F_x^2 + F_y^2} \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$ $F_{Rx} = \sum F_{xi} \quad F_{Ry} = \sum F_{yi}$
Assumptions	What assumptions are being made about the situation (e.g., rigid body, forces are constant, forces in same plane, etc.)
Solution	Show all calculations done to determine the answer.
Verification	Perform the calculation in a different way or from a different reference point.
Conclusion	Provide a summary of the answer and what the answer means for the situation presented in the problem.

EXAMPLE I



Given: Three concurrent forces acting on a tent post.

Find: The magnitude and angle of the resultant force.

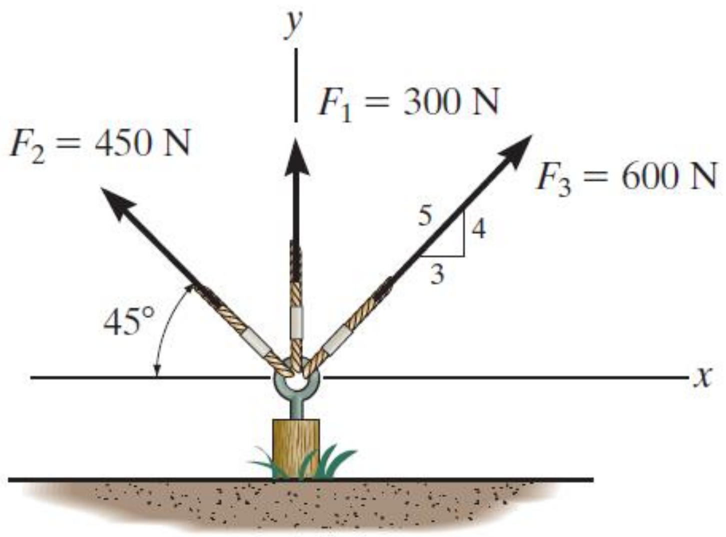
Plan:

a) Resolve the forces into their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

EXAMPLE I (continued)



$\boldsymbol{F_1} = \{0\ _{1x} + 300\ _{1y}\}$ N

$\boldsymbol{F_2} = \{-\ 450 \cos (45°)\ _{2x} + 450 \sin (45°)\ _{2y}\}$ N

$= \{-\ 318.2\ _{2x} + 318.2\ _{2y}\}$ N

$\boldsymbol{F_3} = \{\ (3/5)\ 600\ _{3x} + (4/5)\ 600\ _{3y}\}$ N

$= \{\ 360\ _{3x} + 480\ _{3y}\}$ N

EXAMPLE I (continued)

Summing up all the ***x*** and ***y*** components respectively, we get,

$\boldsymbol{F_R} = \{ (0 - 318.2 + 360)\ \boldsymbol{_{Rx}} + (300 + 318.2 + 480)\ \boldsymbol{_{Ry}} \}$ N

$= \{ 41.80\ \boldsymbol{_{Rx}} + 1098\ \boldsymbol{_{Ry}} \}$ N

Using magnitude and direction:

$F_R = ((41.80)^2 + (1098)^2)^{1/2} = \underline{1099\text{ N}}$

$\phi = \tan^{-1}(1098/41.80) = \underline{87.8°}$